
Numerical Mathematics the Difference method an Babbage use of it

In general during my school education I had the privilege of good teachers. But it happens, and perhaps I am not alone in this, that I have learned on school thinks without knowing the real possible need / practical reasons behind. It could very well be caused by my missing attention, but reading a few stories on the Babbage difference engine and the method Babbage used for his task of building mathematical tables, made me clear why we learned this method. One of the fundamental needs for using numerical mathematics and especially the method Babbage used, the "Difference method", was forced by the limitations of the at that time possible calculation tools.

It is perhaps good to summarize this method first, for those who not knowing where I talk about, and those who might have forgotten it.

Numerical mathematics is given us the results of a function $y=f(x)$ where x is a discrete, equidistance, continuous number range. And the function might be of every order. To solve these functions, mathematical derivations have to be executed, to made the calculation as simple as possible. And that is part of my déjà vu. At the time Babbage invent his machine, this simplifications make sense. The existing calculators were not to a level that they could handle power calculations, even multiplying and divide was a difficulty and programmability was not invented yet. So to simplify the $y=f(x)$ calculation was for sure motivated by the limitations of the calculator machine.

Babbage had given himself the task to create logarithmic, mathematical and trigonometric function tables. Those existing tables were calculated by humans and had failures. The difference method is very well suitable for this task. The method is based on starting with a value of x and calculate the start $f(x)$ and calculate from then on the difference between $f(x+1)$ and $f(x)$. And add this difference to the previous result of $f(x)$.

An example of an simple second order function $f(x)=x^2$.

The difference is $g(x)= f(x+1) - f(x)$

Fill in the function $f(x)=x^2$, then we get de difference $g(x)=x^2 +2x+1 -x^2 = 2x+1$

nice to see is that the difference is of the first order, one order lower than the original function $f(x)$

So what the machine has to calculate is an adding (two times x) + (one) + (the previous result).

An order simpler than the function $f(x)$ itself.

The table looks like this:

x	x^2
0	0
1	$0 + 2 \times 0 + 1 = 1$
2	$1 + 2 \times 1 + 1 = 4$
3	$4 + 2 \times 2 + 1 = 9$
4	$9 + 2 \times 3 + 1 = 9 + 6 + 1 = 16$

So the results are what we expected.